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Intermittency of 1D velocity spatial profiles in turbulence: a magnitude cumulant analysis

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Abstract. We perform one- and two-points magnitude cumulant analysis of one-dimensional longitudinal velocity profiles stemming from three different experimental set-ups and covering a broad range of Taylor scaled Reynolds numbers from $R_\lambda = 89$ to 2500. While the first-order cumulant behavior is found to strongly depend on Reynolds number and experimental conditions, the second-order cumulant and the magnitude connected correlation functions are shown to display respectively universal scale and space-lag behavior. Despite the fact that the Extended Self-Similarity (ESS) hypothesis is not consistent with these findings, when extrapolating our results to the limit of infinite Reynolds number, one confirms the validity of the log-normal multifractal description of the intermittency phenomenon with a well defined intermittency parameter $C_2 = 0.025 \pm 0.003$. But the convergence to zero of the magnitude connected correlation functions casts doubt on the asymptotic existence of an underlying multiplicative cascading spatial structure.

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1 Introduction

Since Richardson's original work [1], a common "mental image" of fully developed turbulence is a dynamical cascading process in which large eddies split up into smaller ones which themselves blow up into even smaller ones and so forth. According to this picture, energy propagates from the integral scale, where eddies are generated, down to the dissipative scale, where they vanish by viscous dissipation, through a multiplicative process, each eddy inheriting a fraction of its parent's energy. Since this early intuitive description, the notion of *cascade* has remained the creed of many models proposed in the literature [2] to mimic the statistical properties of turbulent signals. In 1941, Kolmogorov [3] resumed Richardson's picture in his statistical analysis of the spatial fluctuations of velocity profile in the sense that he linked the one-point statistics of the velocity increments $\delta v_l = v(x+l) - v(x)$ over different distances l, by some dimensional analysis which predicted the remarkable scaling behavior of the moments of δv_l:

$$M(q,l) = \langle \delta v_l^q \rangle \sim l^{\zeta_q}, \qquad (1)$$

where $\zeta_q = q/3$. Actually, ζ_q turned out to be a non-linear function of q in most experiments [2,4–6] and many studies inspired from Kolmogorov and Obukhov second theory [7] tried to explain and to predict the analytical shape of this non-linearity. The understanding of the intermittency phenomenon in fully developed turbulence remains a challenging task not only from a theoretical point of view but also at an empirical level [2]. The controversial situation at the origin of some disagreement between models (*e.g.* the log-normal [7–9] as opposed to log-Poisson models [10]) results from the experimental observation that the moments $M(q,l)$ do not really scale perfectly. Indeed, there is a persistent curvature when one plots $\ln(M(q,l))$ *vs.* $\ln(l)$, which means that, rigorously speaking, there is no scale invariance. In order to give a sense to the exponents ζ_q, Benzi *et al.* [11] defined the "Extended Self-Similarity" (ESS) hypothesis by proposing the following behavior for the moments of the velocity increments :

$$M(q,l) \sim f(l)^{\zeta_q}, \qquad (2)$$

where $f(l)$ would be some q-independent function of l. Along this line, $\ln(M(q_1,l))$ *vs.* $\ln(M(q_2,l))$ curves look definitely more linear than using standard log-scale representations and, by assuming that $\zeta_3 = 1$ [2], some experimental consensus has apparently been reached on the nonlinearity of the ζ_q spectrum [6]. In a recent theoretical work [12], Arad *et al.* suggest that at low Reynolds number, structure functions scaling properties are "polluted" by anisotropic effects that can be mastered using the irreducible representations of the rotation group. Unfortunately, this analysis is not tractable with single point data. Hopefully, as shown below, some statistical quantities turn out to display universal behavior that are likely to be insensitive to anisotropic effects.

In the early nineties, Castaing *et al.* [8] recasted the cascade picture and the ESS hypothesis in a probabilistic

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description that accounts for the continuous deformation of the probability density functions (pdf) of δv_l with l, by mean of a propagator $G_{ll'}$ $(l' > l)$:

$$P_l(\delta v) = \int_{-\infty}^{\infty} G_{ll'}(u) \mathrm{e}^{-u} P_{l'}(\mathrm{e}^{-u}\delta v) \mathrm{d}u. \quad (3)$$

This equation can be related to a cascade process in which the variable δv_l is continuously decomposed as $\delta v_l = \prod_{i=1}^{n} W_{l_{i+1},l_i} \delta v_{l'}$, where $\ln(|W_{l_{i+1},l_i}|)$ are independent random variables of law $G_{l_{i+1}l_i}$. Within this framework, the ζ_q and the $f(l)$ ESS functions can be related to the shape of $G_{ll'}$ thanks to the cumulant generating function of the *magnitude* $\ln|\delta v_l|$ [9,13], *i.e.* the logarithm of the Fourier transform $\hat{G}_{ll'}$ of $G_{ll'}$:

$$\begin{aligned} \ln \hat{G}_{ll'}(-\mathrm{i}q) &= \ln \left(M(q,l)/M(q,l') \right), \\ &= \zeta_q \left(f(l) - f(l') \right), \end{aligned} \quad (4)$$

the classical scale invariant case corresponding to $f(l) = \ln(l)$. In this paradigm, the successive terms of the polynomial development of $\ln(M(q,l)/M(q,l'))$ as a function of q, involve the cumulants $C_n(l) = -C_n f(l)$ of $\ln|\delta v_l|$ (Eq. (7)) from which one can express the ζ_q spectrum as:

$$\zeta_q = -\sum_{n=1}^{+\infty} C_n \frac{q^n}{n!} \, . \quad (5)$$

Let us notice that Castaing's approach can be linked to the recently proposed Fokker-Planck/Langevin description of the intermittency [14]. In these papers, the authors assume that the velocity field is a Markov process across scales and suggest that the velocity increment pdf at different scales obey a Fokker-Planck differential equation characterized by a drift and a diffusion coefficient. Even though this description remains, to a large extent, formal from a mathematical point of view and, very much like the cascade paradigm, very phenomenological, it can be interesting because of its great versatility as far as scaling behavior is concerned [15]. Moreover, some recent works have tried to build some bridge between the Fokker-Planck approach and Navier-Stokes dynamics [16].

The question whether turbulent velocity signals do (or do not) present a space-scale cascade structure thus amounts to checking that the successive velocity magnitude cumulants possess the same scale behavior. This perspective can be extended from structure functions to unfused correlations functions [17] (*e.g.*, $\langle \delta v_l^q(x) \delta v_{l'}^p(x + \Delta x) \rangle$), by studying the behavior of the magnitude connected correlation functions (MCCF):

$$C_{ll'}(\Delta x) = \langle \ln(|\delta v_l(x)|) \ln(|\delta v_{l'}(x+\Delta x)|) \rangle_{\mathrm{c}} \; , \quad (6)$$

where the symbol c stands for "connected" correlations, *i.e.* the derivatives of the 2-points cumulant generating function (see Sect. 3). These correlation functions have been introduced in reference [18] and play the same role for the multivariate multi-point velocity law as the previous magnitude cumulants for the one-point law. If unfused correlation functions are assumed to scale with the lag Δx (as for multiplicative cascade processes), then the MCCF are expected to display a logarithmic dependence on the spatial distance Δx [18].

The goal of this paper is to push further the analysis carried out in references [9,18] by studying both the cumulants and the connected correlation functions associated to $\ln(|\delta v_l|)$ at different scale l, with the aim to establish a clear diagnostic about (i) the scaling properties of experimental turbulent velocity records and (ii) to quantify the intermittent character of the velocity field at different Reynolds numbers. Among the available experimental data, a lot of effort has been devoted to the study of the longitudinal velocity component recorded in directional flows such as jets or wind tunnels [2,4–6]. In these configurations, the Taylor hypothesis that considers the spatial structure of the flow as globally advected upon the probe, enables us to interpret temporal time series as spatial profiles. We will work with signals gracefully supplied by three different experimental groups. The signals labelled with Taylor scaled Reynolds number $R_\lambda = 89$, 208, 463, 703 and 929 stem from Castaing's group and were recorded in a gaseous helium jet at very low temperature in CRTB (Grenoble) [19]; the signal labelled $R_\lambda = 570$ stems from Baudet's group and was recorded in an air jet in LEGI (Grenoble); the signal labelled $R_\lambda = 2500$ was recorded at the ONERA wind tunnel in Modane by Gagne and collaborators [20].

2 One-point magnitude cumulant analysis

Let us start investigating one-point statistics *via* the computation of the cumulants $C_n(l)$ of $\ln(|\delta v_l|)$. This amounts to assume the following general cumulant expansion of structure function exponents:

$$M(q,l) = \langle |\delta v_l|^q \rangle = K_q \, \mathrm{e}^{\sum_{n=1}^{+\infty} C_n(l) \frac{q^n}{n!}} \, . \quad (7)$$

Let us note that the ESS situation (Eq. (2)) corresponds to $C_n(l) = -C_n f(l)$, $\forall n$. Let us also point out that within this framework, the Fokker-Planck model for the even part of the velocity pdf is equivalent to suppose that only the first two cumulants $C_1(l)$ and $C_2(l)$ are non zero and are simply related to the drift and diffusion coefficients [14,15]. The cumulants $C_n(l)$ can be estimated by using a polynomial fit of the curve $\ln(M(q,l)/\ln(M(q,l_0))$, where l_0 is an arbitrary reference scale (let us remark that the cumulants $C_n(l)$ are defined in equation (7) up to an additive constant that can be changed by a redefinition of K_q). In practice, we have estimated the first three cumulants by using polynomial least square fits of order 3 and 4 that turn out to provide the same results. In Figure 1a, $C_1(l)$ is plotted *vs.* $\ln(l)$ for all signals. They are all characterized by an integral scale L corresponding to a decorrelation of the increments $|\delta v_l|$ for $l > L$. Below this saturated regime, there is no well defined "inertial range" since one observes a continuous cross-over across scales towards the smooth dissipative regime down the Kolmogorov scale η. Let us note that the integral scale does not depend on

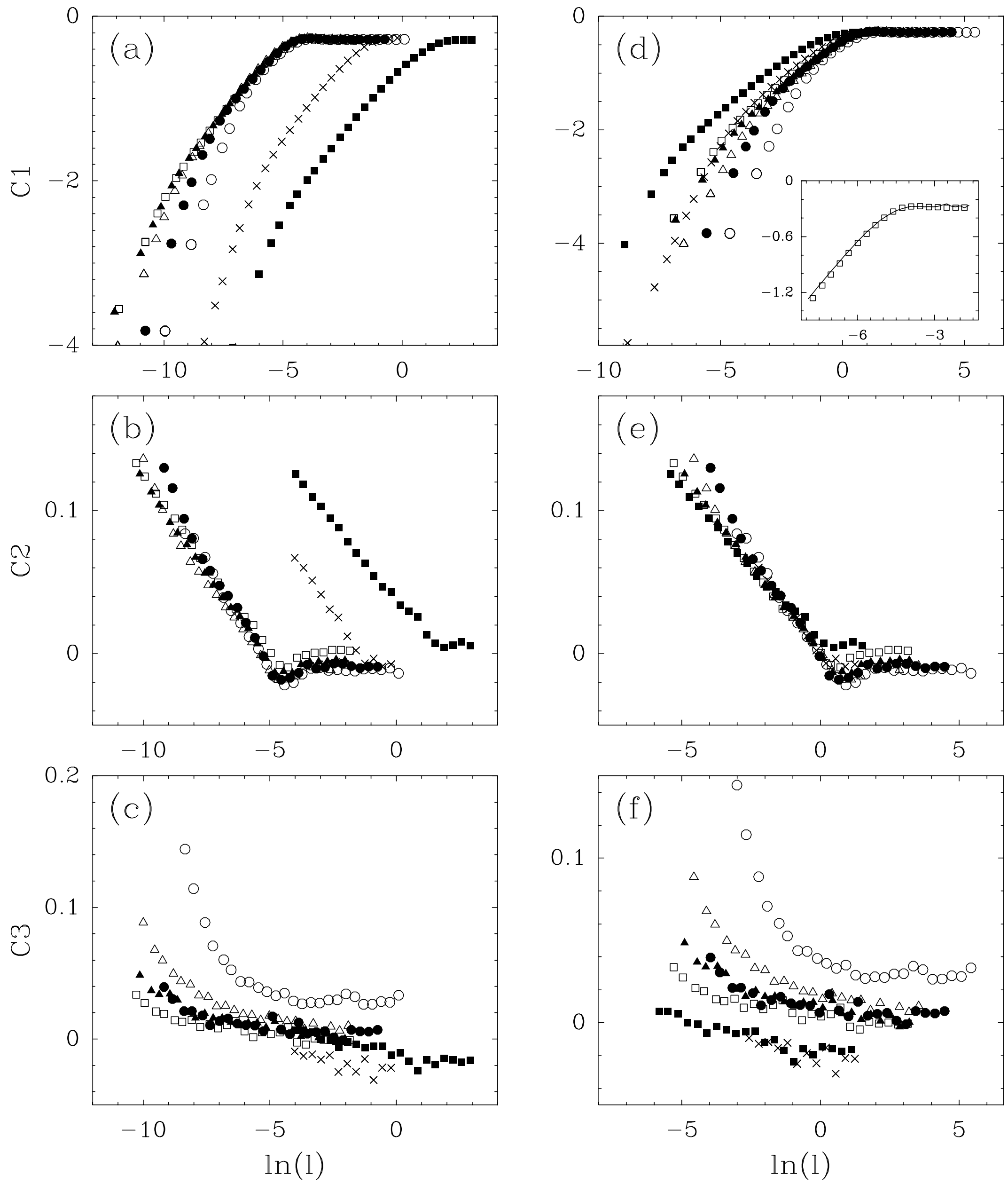

Fig. 1. Magnitude cumulants $C_n(l)$ of the seven studied signals for $R_\lambda = 89$ (○), 208 (•), 463 (△), 570 (×), 703 (▲), 929 (□) and 2500 (■). (a) $C_1(l)$ *vs.* $\ln(l)$: some continuous cross-over is observed from a decorrelated regime at large scales down to a smooth one at small scales. (b) $C_2(l)$ *vs.* $\ln(l)$: a scaling behavior is obtained with the same slope $-C_2 = -0.025 \pm 0.003$ for all signals. (c) $C_3(l)$ *vs.* $\ln(l)$: in the inertial range and for the largest values of R_λ, the slope is compatible to zero up to finite sample effects. (d), (e) and (f): same curves as in (a), (b) and (c), when all integral scales are set to 1. In the inset of (d), the solid line corresponds to the prediction of the Langevin model defined in the text after adjusting the integral scale L and the variance of the noise to the measured ones.

the Reynolds number but only on geometrical considerations. As an illustration, one gets the same value for L for all the signals stemming from Castaing's group which were recorded in the same apparatus, R_λ being controlled either by the mean speed of the flow or by the value of viscosity that is changed by tuning temperature. Furthermore, the numerical values obtained for L: 1.82 cm for Castaing's group, 0.91 m for Baudet's group and 10.2 m for Gagne's group, correspond to the characteristic sizes of the experimental set-ups. L is thus a geometric parameter and when one increases R_λ, the inertial range increases *via* the decrease of η. Let us notice that in reference [19], a Langevin model is proposed as a simple model of the Lagrangian dynamics of a particle in the flow. In this model, a friction constant is introduced to saturate the r.m.s. velocity at large times. By writing the same kind of Langevin equation for Eulerian velocity increments, $\dot{\delta v} = -\gamma \delta v + W(t)$ (where $\gamma = v_{\rm r.m.s}/L$ and W is a white noise whose variance corresponds to the mean dissipation ϵ), and by interpreting the Eulerian second order structure function $\langle \delta v_l^2 \rangle$ as the parametrized curve $\langle \delta v^2(t) \rangle$ *vs.* $l(t) = \langle (\int_0^t v(t')\mathrm{d}t')^2 \rangle^{1/2}$, one can also reproduce the shape of the cross-over of $C_1(l)$ from the saturated regime to the inertial one as shown in the inset of Figure 1d. However this approach does not account for the cross-over in the dissipative region and the precise functional form of $C_1(l)$ is likely to depend on the experimental parameters and to be strongly influenced by anisotropic effects [12] as well as by the inhomogeneity of the flow [21].

The situation is very different for the second magnitude cumulant $C_2(l)$ when plotted *vs.* $\ln(l)$ as shown in Figure 1b. We first notice that the decorrelation scale where $C_2(l)$ saturates, is even better defined with $C_2(l)$ than with $C_1(l)$, since a linear behavior is observed up to this scale without any cross-over. The most important feature is that, whatever the set-up and whatever R_λ, all the data fall on *linear* curves which all have the *same* slope $-C_2 = -0.025 \pm 0.003$. As a matter of fact, if we superimpose all the curves by setting all L to 1, as shown in Figure 1e, we cannot distinguish anymore one curve from the others. This consideration is particularly relevant in the case of the signals of Castaing's group where R_λ is varied without changing L.

The results concerning the third magnitude cumulant $C_3(l)$ are reported in Figures 1c and 1f. The slope of the curves obtained when plotting $C_3(l)$ *vs.* $\ln(l)$ systematically decreases when increasing R_λ. For small R_λ, C_3 is significantly different from zero which means that the log-normal paradigm is not valid. In fact, from the curvature of the corresponding experimental $C_1(l)$ curves, we believe that this is rather a confirmation of the absence of a well defined inertial range for these low values of R_λ. For the largest values of R_λ (≥ 800), C_3 becomes small enough (up to finite sample effects as previously reported in Ref. [9]) to be neglected [22]. At high Reynolds numbers, one can thus suppose that $C_3 = 0$ (and so the higher order cumulants) that implies a normal shape for the propagator $G_{ll'}$ in agreement with log-normal models [7–9] and with the findings of references [14] justifying a Fokker-Planck description. To summarize, the scale behavior of structure functions is well described by the first two terms in the cumulant expansion of $G_{ll'}$ and the differences observed in the behavior of $C_1(l)$ (Figs. 1a and d) and $C_2(l)$ (Figs. 1b and e) bring the experimental demonstration of the inconsistency of the ESS hypothesis which rigorously requires an identical scale behavior for the two cumulants (Eq. (7)). The full ζ_q spectrum cannot thus be defined in the range of Reynolds numbers we have investigated but the scaling exponent $C_2 = 0.025 \pm 0.003$ of $C_2(l)$ appears to be a universal (model independent) characteristic of intermittency.

3 Two-point magnitude cumulant analysis

Let us now focus on the MCCF, $C_{ll'}(\Delta x)$ defined in equation (6). This function is of central interest if one extends the previous log-normal hypothesis to the whole magnitude process (this assumption will be explicitly checked in a forthcoming paper). In that case, the unfused n-points correlation function

$$M(q_1 \ldots q_n; l_1 \ldots l_n; x_1 \ldots x_n) = \langle |\delta v_{l_1}(x_1)|^{q_1} \ldots |\delta v_{l_n}(x_n)|^{q_n}| \rangle, \quad (8)$$

where $|x_i - x_j| > \sup(l_i, l_j)$, can be simply expressed in terms of the structure functions $M(q_i, l_i)$ and the MCCF $C_{l_i l_j}(x_i - x_j)$ as:

$$M(q_1 \ldots q_n; l_1 \ldots l_n; x_1 \ldots x_n) = \prod_{i=1}^{n} M(q_i, l_i) \mathrm{e}^{\frac{1}{2}\sum_{i \neq j} q_i q_j C_{l_i l_j}(x_i - x_j)}. \quad (9)$$

Let us recall that for scale invariant processes and particularly for a cascade process, we expect the scaling behavior

$$M(q_1 \ldots q_n; \lambda l_1 \ldots \lambda l_n; \lambda x_1 \ldots \lambda x_n) \sim \lambda^{h(q_1 \ldots q_n)} M(q_1 \ldots q_n; l_1 \ldots l_n; x_1 \ldots x_n), \quad (10)$$

which implies that the MCCF is a logarithmic function:

$$C_{ll'}(\Delta x) = -C_2 \ln(\Delta x / L). \quad (11)$$

In Figure 2 are shown the MCCF computed at a single small scale ($l = l'$) for each experimental signal. Three main observations must be raised from these curves. First, the MCCF do not behave linearly but rather as the square of the logarithm of the spatial distance Δx:

$$C_{ll}(\Delta x) = \alpha(R_\lambda) \ln^2(\Delta x / L) \; . \quad (12)$$

This quadratic behavior is universal as far as experimental set-ups are concerned and has never been observed before. Second, the respective integral scales at which the magnitudes are decorrelated are nearly the same as the ones observed on the corresponding cumulants $C_n(l)$ ($n =$ 1,2), which is not a trivial result. As pointed out in reference [18] and illustrated in Figure 2b, one can check that the MCCF computed at different scales l and l' all collapse on a single curve provided $\Delta x > \max(l, l')$. Finally, as shown in Figure 3, the prefactor $\alpha(R_\lambda)$ in equation (12) has a systematic decreasing behavior as a function of R_λ. As far as the analytic shape of this decrease is concerned, there is no certainty. As illustrated in Figure 3a, a power-law behavior: $\alpha \sim R_\lambda^{-a}$ with $a \simeq 0.20 \pm 0.03$ provides a reasonable good fit of the data. But as shown in Figure 3b, one can also fit the data by $\alpha \sim K / \ln(R_\lambda)$ with $K = 0.027 \pm 0.006$, which is a particular Reynolds number dependance that allows us to define a characteristic scale of the flow under study. Indeed, if one considers that the decorrelation scale is the same for the magnitude cumulants $C_n(l)$ and the MCCF, from the behavior of the variance as $-C_2 \ln(l/L)$ and of the MCCF as $\alpha \ln^2(\Delta x / L)$, one can define a characteristic scale l_c where the two curves meet (when identifying scale and space-lag according to the multiplicative

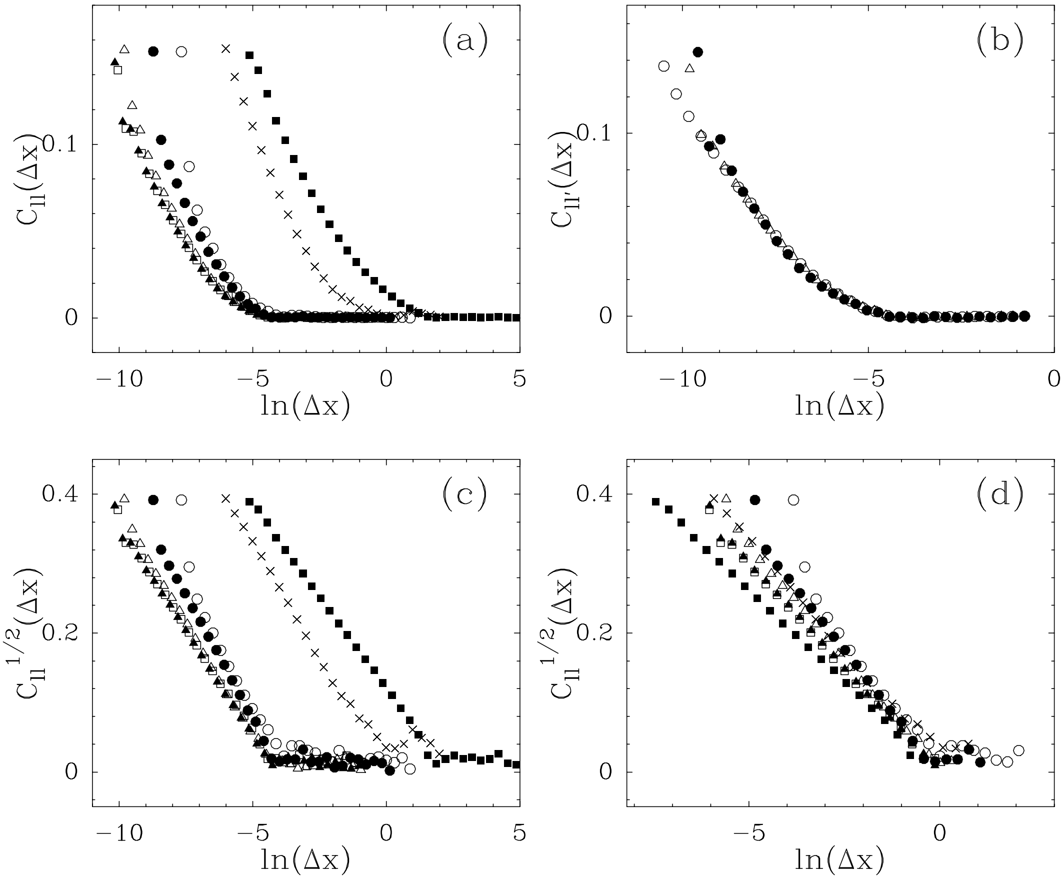

Fig. 2. Magnitude connected correlation functions. (a) $C_{ll}(\Delta x) = \langle \ln(|\delta v_l(x)|)\ \ln(|\delta v_l(x+\Delta x)|)\rangle_{\rm c}$ *vs.* $\ln(\Delta x)$. (b) $C_{ll}(\Delta x)$, $C_{l'l'}(\Delta x)$ and $C_{ll'}(\Delta x)$ for $l' = 8l$. (c) Square root of $C_{ll}(\Delta x)$ *vs* $\ln(\Delta x)$; $C_{ll}(\Delta x)$ behaves as $\alpha \ln^2(\Delta x)$ whatever the experimental set-up with a systematic decrease of α when R_λ is increased. (d) Same curves when all integral scales L are set to 1. The considered scale is $l \simeq 50\ \mu$m for Castaing's signals, 0.15 mm for Baudet's signal and 3.2 mm for Gagne's signal. The symbols correspond respectively to the seven same signals as in Figure 1.

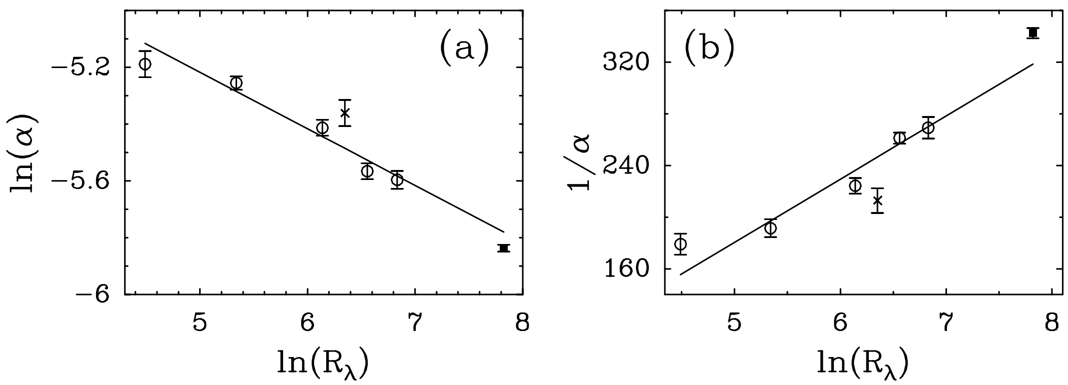

Fig. 3. Prefactor $\alpha(R_\lambda)$ of the quadratic behavior of $C_{ll}(\Delta x)$ *vs.* $\ln(\Delta x)$ (see Fig. 2). (a) $\ln(\alpha)$ *vs.* $\ln(R_\lambda)$; the data are compatible with a power-law behavior $\alpha \sim R_\lambda^{-a}$ with $a = 0.20 \pm 0.03$ (solid line). (b) $1/\alpha$ as a function of $\ln(R_\lambda)$; the data are compatible with a behavior $\alpha \sim K/\ln(R_\lambda)$ with $K = 0.027 \pm 0.006$ (solid line). The symbols have the following meaning: ($\circ$) Castaing's group, ($\times$) Baudet's group and (■) Gagne's group.

cascade picture [18]): $\ln(l_c/L) = -C_2/\alpha$. With the numerical values extracted from Figures 1b, 2 and 3b, one gets $(l_c/L) \sim R_\lambda^{-b}$ with $b = 0.91 \pm 0.3$. This new scale l_c might well be the Taylor scale λ for which $b = 1$. In that respect, our approach could be a way to measure objectively λ and this even when the dissipative scale η is not resolved. This universal quadratic behavior of the MCCF *vs.* $\ln(\Delta x)$ and the systematic decrease of $\alpha(R_\lambda)$ when increasing R_λ, are new observations which enables us to make some conjecture about the asymptotic limit of infinite Reynolds number. As pointed out in references [9,18], correlations in the magnitude are the signature of a multiplicative structure. If we extrapolate the observed behavior to infinite R_λ, we predict the convergence to zero of the MCCF on any finite inertial range of scales. That is to say, in the limit of infinite R_λ, the spatial fluctuations of the longitudinal velocity component are, from the point of view of unfused correlation functions, not likely to possess any intermittent nature, which contrasts with the previous conclusions based on one-point statistics.

4 Conclusion

To summarize, we have advocated in this paper the study of scaling properties of longitudinal velocity increments by means of one- and two-points magnitude cumulant analysis. The results of our measurements for seven flows, at seven different R_λ and stemming from different experimental set-ups, are two-fold. Concerning the one-point (fused) statistics, we mainly observe that the log-normal approximation is pertinent at sufficiently high Reynolds number and that the first two cumulants have not the same scale behavior as assumed by the ESS hypothesis. While $C_1(l)$ displays some Reynolds and set-up dependent departure from scale invariance, $C_2(l)$ exhibits scale invariance behavior with a universal intermittency coefficient $C_2 = 0.025 \pm 0.003$. Concerning the two-points (unfused) statistics, we observe that the behavior of the MCCF *vs.* $\ln(\Delta x)$ is quadratic whatever the considered experiment but with a prefactor $\alpha(R_\lambda)$ which decreases when increasing R_λ. These new observations lead us to conjecture that in the limit of infinite Reynolds number, the multipoint statistics of longitudinal velocity fluctuations are not intermittent. Accordingly the intermittency, as well defined by the intermittency exponent, appears only in the fused statistics in the spirit of the log-normal multifractal description pioneered by Kolmogorov and Obukhov in 1962 [7], *i.e.* with an intermittency coefficient $C_2 = 0.025 \pm 0.003$ but without any correlations across scales. This is not at all shocking from a physical point of view since the dynamical cascading process [2] does not *a priori* imply that there should be some multiplicative (cascading) spatial organization. Furthermore, one must realize that our conclusions come out from the study of 1D cuts of the velocity field only. In a forthcoming publication, we plan to carry out a similar analysis of a 3D velocity field issued from direct numerical simulations, with the specific goal of testing the validity of this non multiplicative log-normal multifractal picture to account for the intermittent nature of fully developed turbulent 3D velocity fields.

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22. Note that statistical convergence is faster in the estimation of C_3 when using the propagator as in reference [9] than when computing directly $C_3(l)$ as reported in Figure 1c.